

$D = 2$ gluon condensate and QCD propagators at finite temperature

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Abstract

We calculate the dimension two gluon condensate contribution to quark, gluon and ghost propagators at finite temperature.

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Quantum Chromodynamics (QCD) has been firmly established as the correct theory of strong interaction. At high enough temperature, the QCD matter undergoes a rapid crossover [1] from confined hadronic phase to a quasifree gas of deconfined quarks and gluons called quark gluon plasma [2]. The structure of QCD near the phase transition seems to be rather complex and dominated by strong non-perturbative effects [3] from the infrared sector of the theory. These non-perturbative effects are parameterized by non-vanishing local condensates, such as $D = 3$ quark condensate $\langle \bar{\psi}\psi \rangle$ and $D = 4$ gluon condensate $\langle G_{\mu\nu}^2 \rangle$. The quark condensate is related to the spontaneous breaking of chiral symmetry while the gluon condensate is the manifestation of the broken scale invariance. Through the operator product expansion (OPE), these condensates appear with power corrections in physical observables which provide non-perturbative information in addition to perturbatively calculable radiative corrections. This strategy has met with success in QCD sum rule calculations both at zero [4] and non-zero temperature [5]. The operator product expansion has also been used to investigate nonperturbative corrections in N point Green's functions at zero temperature [6] as well as at finite temperature [7–10].

Of late, there has been much interest on the existence of a dimension two BRST invariant gluon condensate $\langle A^2 \rangle$ [11–15]. This operator is not gauge invariant and does not show up in QCD sum rule calculations which deal with gauge invariant correlation functions. However, the situation is different when one considers gauge variant quantities such as propagator. In principle, any condensate with appropriate quantum numbers may appear in the OPE of Green's functions [16]. One can formally construct a gauge invariant, albeit non-local, $\langle A^2 \rangle$ by averaging it over all possible gauge transformations. A non-local but gauge invariant operator does not make much sense from the perspective of operator product expansion. Alternatively one may evaluate the operator in a specific gauge. In particular, it has been shown that the bulk averaged $\langle A^2 \rangle$ attains a minimum in Landau gauge. Lattice simulations show that $\langle A^2 \rangle$ condensate in Landau gauge at zero temperature is in fact a large quantity [11, 17, 18].

At finite temperature, the temporal (electric) and spatial (magnetic) components of the gauge field fluctuations have different size in general. In fact, the scenario of dynamical generation of a $\langle A_0^2 \rangle_T$ condensate at finite temperature has been around in the literature for some time. Possible phenomenological implication of such a scenario have been discussed in the context of static screening of chromo-electric and chromo-magnetic fields [19, 20] (see

also [21] and references therein), dilepton production rate [22], Polyakov loop correlator [23], trace anomaly [24] or physics of heavy quarks [25, 26] in the quark gluon plasma. Nonzero values of electric $\langle A_0^2 \rangle_T$ and $\langle A_i^2 \rangle_T$ have been confirmed on lattice [27, 28] for $SU_c(2)$. So far no lattice measurements for these condensates have been performed for $SU(3)$ but it is expected to be qualitatively different.

With the understanding that $D = 2$ gluon condensates have important role in the non-perturbative dynamics of glue matter, we aim in this letter to explicitly evaluate their contribution to the in-medium quark, gluon and ghost propagation above the deconfinement temperature.

I. NON PERTURBATIVE GLUON PROPAGATOR AND GLUON CONDENSATES

In QCD, polarization tensor is not transverse in general, $K^\mu \Pi_{\mu\nu}(K) \neq 0$. For an $O(3)$ invariant gauge fixing condition, the most general tensorial structure of the in-medium gluon self-energy can be written as (we omit trivial color factor) [29]

$$\pi_{\mu\nu}(\omega, k) = \pi_l(\omega, k) \mathcal{P}_{\mu\nu}^l + \pi_t(\omega, k) \mathcal{P}_{\mu\nu}^t + \pi_m(\omega, k) \mathcal{M}_{\mu\nu} + \tilde{\pi}(\omega, k) l_{\mu\nu}. \quad (1)$$

Here, ω and k are the Lorentz invariant single particle energy and momentum respectively,

$$\begin{aligned} \omega &= u \cdot K, \\ k &= [(u \cdot K)^2 - K^2]^{\frac{1}{2}}, \end{aligned} \quad (2)$$

and u^μ is the four velocity of the heat bath. In the rest frame of the medium $u^\mu = \delta_0^\mu$. The projection operators are defined as [29, 30],

$$\mathcal{P}_{\mu\nu}^l = -\frac{1}{k^2 K^2} \left(k^2 u_\mu + \omega \widetilde{K}_\mu \right) \left(k^2 u_\nu + \omega \widetilde{K}_\nu \right) = \frac{K^2}{\widetilde{K}^2} \bar{u}^\mu \bar{u}^\nu, \quad (3a)$$

$$\mathcal{P}_{\mu\nu}^t = \eta_{\mu\nu} - u_\mu u_\nu - \frac{\widetilde{K}_\mu \widetilde{K}_\nu}{\widetilde{K}^2}, \quad (3b)$$

$$\mathcal{M}_{\mu\nu} = -\frac{1}{\sqrt{-2\widetilde{K}^2}} (\bar{u}_\mu K_\nu + \bar{u}_\nu K_\mu), \quad (3c)$$

$$l_{\mu\nu} = \frac{K_\mu K_\nu}{K^2}. \quad (3d)$$

where $\widetilde{K}_\mu = K_\mu - \omega u_\mu$ and $\bar{u}^\mu = u^\mu - \frac{\omega}{K^2} K^\mu$. The projectors $\mathcal{P}_l^{\mu\nu}$ and $\mathcal{P}_t^{\mu\nu}$ are transverse with respect to K^μ and $\mathcal{M}^{\mu\nu}$ satisfies a weaker condition $K_\mu \mathcal{M}^{\mu\nu} K_\nu = 0$. The scalar structure

FIG. 1. Graphical representation of Eq. 8. Full and nonperturbative propagators are denoted by circular and oval blobs respectively.

functions in the self energy are extracted through appropriate projections,

$$\pi_l = \mathcal{P}_l^{\mu\nu} \pi_{\mu\nu}, \quad (4a)$$

$$\pi_t = \frac{1}{2} \mathcal{P}_t^{\mu\nu} \pi_{\mu\nu}, \quad (4b)$$

$$\pi_m = -\mathcal{M}^{\mu\nu} \pi_{\mu\nu}, \quad (4c)$$

$$\tilde{\pi} = l^{\mu\nu} \pi_{\mu\nu}. \quad (4d)$$

Now from (1), the most general form of the gluon propagator $\mathcal{D}_{\mu\nu} = \mathcal{D}_{0,\mu\nu} (1 + \pi_{\mu\nu} \mathcal{D}_{0,\mu\nu})^{-1}$ can be written as,

$$\begin{aligned} \mathcal{D}_{\mu\nu} = & -\frac{\mathcal{P}_{\mu\nu}^t}{K^2 - \pi_t} - \frac{2}{2(K^2 - \pi_l)(\xi^{-1}K^2 - \tilde{\pi}) + \pi_m^2} \\ & \times [(\xi^{-1}K^2 - \tilde{\pi}) \mathcal{P}_{\mu\nu}^l + \pi_m \mathcal{M}_{\mu\nu} + (K^2 - \pi_l) l_{\mu\nu}]. \end{aligned} \quad (5)$$

In covariant gauges, the Slavnov Taylor identity reads $K^\mu \mathcal{D}_{\mu\nu} K^\nu = K^\mu \mathcal{D}_{0,\mu\nu} K^\nu = -\xi$, we get

$$\frac{2(K^2 - \pi_l) K^2}{2(K^2 - \pi_l)(\xi^{-1}K^2 - \tilde{\pi}) + \pi_m^2} = \xi. \quad (6)$$

For $\xi \neq 0$. Eq. (6) can be written as,

$$(K^2 - \pi_l) \tilde{\pi} = \frac{\pi_m^2}{2} \quad (7)$$

This leaves three independent components in (5). So the most general form of the ‘non perturbative’ gluon propagator should be,

$$\begin{aligned} D_{\mu\nu}^{ab} & \stackrel{\text{def}}{=} \mathcal{D}_{\mu\nu}^{ab} - \mathcal{D}_{\mu\nu}^{ab,\text{pert}} \\ & = P_{\mu\nu}^l D_l(\omega, k) + P_{\mu\nu}^t D_t(\omega, k) + \mathcal{M}_{\mu\nu} D_m(\omega, k), \end{aligned} \quad (8)$$

in an obvious notation. Note that, D_m is absent in covariant gauges if 1) $\xi = 0$ (Landau gauge) or 2) the self energy is transverse $\pi_m = \tilde{\pi} = 0$.

Condensates of various dimensions can be related to the moments of the non-perturbative gluon propagator. In the rest frame of the medium ($u^\mu = \delta_0^\mu$), the dimension two condensates are given by,

$$\langle A_0^2 \rangle_T = -T (N_c^2 - 1) \int \frac{d^3 k}{(2\pi)^3} D_l(0, k) , \quad (9a)$$

$$\langle A_i^2 \rangle_T = 2T (N_c^2 - 1) \int \frac{d^3 k}{(2\pi)^3} D_t(0, k) , \quad (9b)$$

where N_c is the number of color. We have restricted here to the lowest Matsubara mode ($k_0 = 0$) in the spirit of planewave method [7, 8].

II. QUARK SELF ENERGY

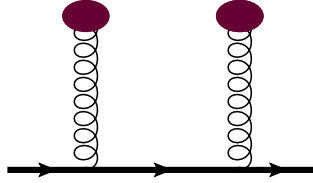


FIG. 2. Gluon condensate contribution to quark self energy.

The $D = 4$ gluon condensate contribution to the quark self energy has been studied in [7]. Here we shall evaluate the $D = 2$ condensate contribution. The general expression for in-medium fermion self energy in the chiral limit is given by,

$$\Sigma(P) = -a(\omega, p) \not{P} - b(\omega, p) \not{\psi} , \quad (10)$$

where the scalar functions are given by,

$$a(\omega, p) = \frac{1}{4p^2} [\text{Tr}(\not{P}\Sigma) - \omega \text{Tr}(\not{\psi}\Sigma)] , \quad (11a)$$

$$b(\omega, p) = \frac{1}{4p^2} [P^2 \text{Tr}(\not{\psi}\Sigma) - \omega \text{Tr}(\not{P}\Sigma)] . \quad (11b)$$

Using the plane wave method [7, 31] and nonperturbative propagator from Eq. (8), we get from Fig. 2

$$a(p_0, p) = -\frac{2\pi\alpha_s}{N_c P^2} \left[\frac{1}{3} \langle A_i^2 \rangle_T - \langle A_0^2 \rangle_T \right] , \quad (12a)$$

$$b(p_0, p) = -\frac{2\pi\alpha_s p_0}{N_c P^2} \left[2 \langle A_0^2 \rangle_T + \frac{2}{3} \langle A_i^2 \rangle_T \right] . \quad (12b)$$

Note that, a and b contain nonperturbative contribution from $D = 4$ gluon condensate and other higher dimensional condensates and perturbative contribution given by HTL corrections as,

$$\begin{aligned} a &= a\langle A^2 \rangle + a\langle G^2 \rangle + \dots + a^{\text{HTL}}, \\ b &= b\langle A^2 \rangle + b\langle G^2 \rangle + \dots + b^{\text{HTL}}. \end{aligned} \quad (13)$$

Then, the chiral quark propagator $S^{-1}(P) = \not{P} - \Sigma$ follows as

$$S(p_0, p) = \frac{\gamma_0 - \gamma \cdot \hat{p}}{2D_+(p_0, p)} + \frac{\gamma_0 + \gamma \cdot \hat{p}}{2D_-(p_0, p)}, \quad (14)$$

where,

$$D_{\pm}(p_0, p) = (-p_0 \pm p)(1 + a) - b. \quad (15)$$

III. GLUON SELF ENERGY

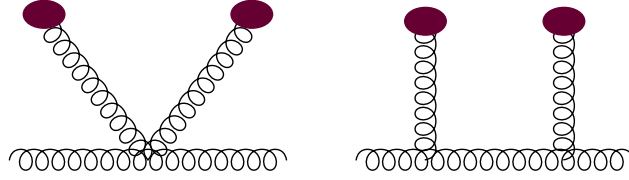


FIG. 3. Gluon self energy with non-perturbative gluon propagator.

The gluon self energy with $D = 2$ gluon condensate follows from Fig. 3. Various scalar structure functions in the self energy as given in (4) are obtained as,

$$\pi_l^{\langle A^2 \rangle} = f_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - f_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T, \quad (16)$$

$$\pi_t^{\langle A^2 \rangle} = g_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - g_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T, \quad (17)$$

$$\pi_m^{\langle A^2 \rangle} = \sqrt{2} h_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - \sqrt{2} h_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T, \quad (18)$$

$$\tilde{\pi}^{\langle A^2 \rangle} = w_i \frac{\alpha_s}{\pi} \langle A_i^2 \rangle_T - w_0 \frac{\alpha_s}{\pi} \langle A_0^2 \rangle_T. \quad (19)$$

with

$$\begin{aligned} f_0(p_0, p) &= \frac{4\pi^2 N_c}{(N_c^2 - 1)} \left\{ \left(-\frac{\vec{p}^2}{P^2} + \frac{4p_0^2}{P^2} + \frac{p_0^2 \vec{p}^2}{P^4} - \frac{4p_0^4}{P^4} - \frac{3p_0^4}{\vec{p}^2 P^2} + \frac{3p_0^6}{\vec{p}^2 P^4} \right) \right. \\ &\quad \left. + \chi \left(\frac{\vec{p}^2}{P^2} - \frac{2p_0^2}{P^2} - \frac{2p_0^2 \vec{p}^2}{P^4} + \frac{4p_0^4}{P^4} + \frac{p_0^4}{\vec{p}^2 P^2} + \frac{p_0^4 \vec{p}^2}{P^6} + \frac{2p_0^6}{P^6} \right) \right\} \end{aligned}$$

$$\left. -\frac{2p_0^6}{\vec{p}^2 P^4} + \frac{p_0^8}{\vec{p}^2 P^6} \right) \Bigg\} , \quad (20a)$$

$$f_i(p_0, p) = \frac{2\pi^2 N_c}{(N_c^2 - 1)} \left\{ \left(-\frac{2\vec{p}^2}{P^2} + \frac{2p_0^2}{P^2} - \frac{4p^4}{3P^4} + \frac{4p_0^2 \vec{p}^2}{P^4} - \frac{8p_0^4}{3P^4} \right) + \chi \left(\frac{p_0^2 \vec{p}^2}{P^4} + \frac{p_0^2 p^4}{3P^6} \right. \right. \\ \left. \left. - \frac{2p_0^4}{P^4} - \frac{5p_0^4 \vec{p}^2}{3P^6} + \frac{7p_0^6}{3P^6} + \frac{p_0^6}{\vec{p}^2 P^4} - \frac{p_0^8}{\vec{p}^2 P^6} \right) \right\} , \quad (20b)$$

$$g_0(p_0, p) = \frac{4\pi^2 N_c}{(N_c^2 - 1)} \left\{ \left(-1 + \frac{11p_0^2}{2P^2} \right) + \chi \left(-\frac{p_0^2}{P^2} + \frac{p_0^4}{2P^4} - \frac{p_0^4}{2\vec{p}^2 P^2} - \frac{p_0^6}{2\vec{p}^2 P^4} \right) \right\} \quad (21a)$$

$$g_i(p_0, p) = \frac{2\pi^2 N_c}{(N_c^2 - 1)} \left\{ \left(\frac{5}{2} + \frac{\vec{p}^2}{P^2} - \frac{11p_0^2}{3P^2} \right) + \chi \left(\frac{\vec{p}^2}{6P^2} + \frac{p_0^2 \vec{p}^2}{6P^4} + \frac{2p_0^4}{3P^4} - \frac{p_0^4}{2\vec{p}^2 P^4} + \frac{p_0^6}{2\vec{p}^2 P^4} \right) \right\} \quad (21b)$$

$$h_0(p_0, p) = \frac{4\pi^2 N_c}{(N_c^2 - 1)} \left\{ -\frac{p_0}{|\vec{p}|} + \frac{p_0^3}{|\vec{p} P^2|} \right\} , \quad (22a)$$

$$h_i(p_0, p) = \frac{2\pi^2 N_c}{(N_c^2 - 1)} \left\{ \frac{p_0}{|\vec{p}|} + \frac{p_0 |\vec{p}|}{3P^2} - \frac{p_0^3}{|\vec{p} P^2|} \right\} , \quad (22b)$$

$$w_0(p_0, p) = 0 , \quad (23a)$$

$$w_i(p_0, p) = 0. \quad (23b)$$

Here $\chi = 1 - \xi$ and ξ is gauge parameter.

At zero temperature, $k^\mu \pi_{\mu\nu}^{\langle A^2 \rangle} = 0$. We find that the transversality is weaker at finite temperature $k^\mu \pi_{\mu\nu}^{\langle A^2 \rangle} k^\nu = 0$. The $D = 4$ condensate contribution to the gluon self energy will be presented elsewhere [32].

IV. GHOST SELF ENERGY

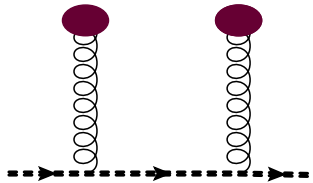


FIG. 4. Gluon condensate contribution to ghost self energy

Similarly, $D = 2$ gluon condensate contribution to ghost self energy in covariant gauge can be evaluated from Fig. 4 as,

$$\Pi(p_0, p) = \frac{4\pi\alpha_s N_c}{N_c^2 - 1} \left[\frac{p_0^2}{P^2} \langle A_0^2 \rangle_T - \frac{1}{2} \left(\frac{4}{3} - \frac{p_0^2}{P^2} \right) \langle A_i^2 \rangle_T \right]. \quad (24)$$

There is no HTL correction for ghost, but ghost can receive non-perturbative mass corrections in a background characterized by non-vanishing condensates. One can similarly calculate Wilson coefficients of $D = 4$ gluon condensates to ghost self energy. To fix these coefficients uniquely, one has to go beyond one loop and calculate Wilson coefficients of $D = 4$ ghost-anti ghost ($\bar{\eta}\square\eta$), and mixed ghost-gluon condensate ($f^{abc}\partial_\mu\bar{\eta}^a A^{\mu,b}\eta^c$). This is similar to gluon self energy calculation at one loop [8] and details will be presented elsewhere [32].

V. SUMMARY

We have calculated $D = 2$ gluon condensate contribution to quark, gluon and ghost self energies at finite temperature. With the values of condensates taken as input from lattice QCD, one can quantitatively predict the analytical structure of these propagators near the transition temperature. As correlated applications, this will be a good starting point to estimate nonperturbative dilepton rate, transport properties of both heavy and light quarks etc. in the deconfined system of quark-gluon matter.

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